A Further Look at the Propagation Mechanism of Monetary Policy Shocks in HANK *

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Abstract

We provide quantitative guidance on the relative importance of Heterogeneous Agent New Keynesian (HANK) model elements for amplification and dampening of the response of aggregate consumption to a monetary shock. We emphasize four findings. First, the presence of capital adjustment costs does not affect the aggregate response, but does change the transmission mechanism so that a larger share of indirect effects come from equity prices rather than labor income. Second, incorporating estimated unequal incidence functions for aggregate labor income fluctuations leads to a small degree of either amplification or dampening, depending on the data and estimation methods. Third, distribution rules for monopoly profits that allocate a larger share to liquid assets lead to greater amplification. Fourth, assumptions about the fiscal reaction to a monetary policy shock have a larger effect on the aggregate consumption response than any of these three elements.

JEL Codes: D14, D31, E21, E52.

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1 Introduction

A recent literature that incorporates micro heterogeneity into New Keynesian models of the macroeconomy has advanced our understanding of the transmission mechanism of monetary policy. In these Heterogeneous Agent New Keynesian (HANK) models, the general equilibrium effects of an interest rate cut, which operate through an increase in household incomes from higher labor demand, outweighs the direct effects of an interest rate cut, which primarily operate through intertemporal substitution. This pattern of transmission stands in stark contrast to the Representative Agent New Keynesian (RANK) models that served as a point of departure for this literature, in which monetary policy affects aggregate consumption almost exclusively through intertemporal substitution and in which indirect effects are negligible.

However, less is currently understood about whether the overall consumption response to an interest rate cut is higher or lower in HANK models than in analogous RANK models. Different versions or parameterizations of seemingly similar HANK models can give different aggregate consumption responses to the same size shock (Werning (2015), Kaplan, Moll, and Violante (2018), Bilbiie (2008)). One reason for the lack of consensus is that the HANK framework incorporates several elements that are either inconsequential or are not well-defined in the RANK framework. Three such elements that we study in this paper include the unequal incidence of aggregate fluctuations across households and the distribution of firm profits. Additional model elements include the cyclicality of idiosyncratic risk, household borrowing capacity and asset liquidity.¹ Depending on their parameterization, these features can either amplify or dampen the effect of a rate cut in HANK, while leaving the effect of rate cut in RANK unchanged. To paraphrase Sims (1980), once we depart from the representative household, we enter the "wilderness of heterogeneous agent macro."

In an attempt to tame this wilderness, a growing literature has used stylized versions of HANK models, that can be solved analytically, to provide theoretical guidance on the model features that determine the extent of propagation (see, for example Auclert, 2016; Werning, 2015; Acharya and Dogra, 2018; Bilbiie, 2017; Debortoli and Gali, 2018; Patterson, 2018). But despite clarifying the channels through which HANK model elements contribute to amplification and dampening, little is currently known about which elements are quantitatively important departures from RANK models, nor whether the insights form these simple models carry through to empirically relevant versions of HANK models.

In this paper, we address this gap by providing quantitative guidance on the rela-

¹See Patterson (2018), Auclert and Rognlie (2018), Bilbiie (2017), Kaplan, Moll, and Violante (2018), Bayer, Luetticke, Pham-Dao, and Tjaden (2015).

tive importance of these candidate propagation mechanisms of monetary shocks in the quantitative HANK model studied by Kaplan, Moll, and Violante (2018). There are two main contributions. First, we parameterize these potential amplification mechanisms in ways that are amenable for quantitative analysis, by providing functional forms with empirically disciplined parameters that can be used as model inputs. Second, we assess the extent to which the lessons from these simple analytical HANK models carry over to our richer model and compare the degree of amplification or dampening across these different model elements.

We pay particular attention to the parameterization and estimation of various "incidence functions" – a concept that has been used by Werning (2015), Auclert and Rognlie (2018) and Patterson (2018). An incidence function describes a rule for how a time-varying aggregate quantity is allocated across the distribution of households in the economy. Building on this existing work, we first propose a convenient parameterization for a general class of incidence functions. We then estimate separate incidence functions for labor income and government transfer income, using various sources of micro data for the United States: the Annual Social and Economic (ASEC) supplement of the Current Population Survey (CPS), the Survey of Consumer Finances and tabulated statistics from the Master Earnings File of the Social Security Administration (SSA).

Additionally, we use the opportunity to examine the consequences of incorporating two model elements that have been shown to be important in RANK models but that were abstracted from in the Kaplan, Moll, and Violante (2018) model: (i) aggregate capital adjustment costs, which helps to generate pro-cyclical asset price movements; and (ii) the presence of an output gap term in the Taylor rule for the nominal interest rate. We examine the effects of these features for both the overall amplification of monetary shocks and the decomposition into direct and indirect effects.

We obtain four main results. First, including aggregate capital adjustment costs in the model does not affect the overall response of aggregate consumption, but does change the transmission mechanism of an interest rate cut. With capital adjustment costs, investment rises by less following a rate cut, and asset prices rise by more. The smaller increase in investment leads to a smaller increase in labor demand and household labor income. Hence a larger share of the general equilibrium increase in consumption is due to the increase in asset prices, and a smaller share due to the increase in labor income. Since wealthier households holds are relatively more exposed to asset prices than poorer households (and relatively less exposed) to labor income, it follows that an expansionary monetary shock redistributes more towards wealth households in the presence of capital adjustment costs.

Second, incorporating unequal incidence of aggregate fluctuations across households into the model, can either amplify or dampen the effects of a monetary policy shock, depending on the specific estimates. For example, estimates using ASEC data suggest that households with low permanent income and higher marginal propensities to consume (MPCs) are the most heavily exposed to fluctuations in aggregate labor income. This leads to an amplification of the aggregate consumption response to a rate cut of up to ten percent, relative to a model with equal incidence. In contrast, estimates using SSA data *dampen* the effect of a rate cut, relative to an equal incidence benchmark. This is because the SSA data suggest that the elasticity of household earnings to aggregate income is U-shaped, with heavily exposed households at both the bottom and the top of the distribution.² There are therefore two offsetting forces at work. More exposure at the bottom, where MPCs are high, leads to amplification relative to equal incidence, but more exposure at the top, where MPCs are low, leads to dampening. Quantitatively, we find that the dampening effects of the low-MPC households dominate, and the net effect is a slight dampening.³ We find that the amplification or dampening effects of unequal incidence are more muted in the presence of capital adjustment costs, because the smaller response of labor income explained above.

Third, in our two-asset model with liquid and illiquid assets, it matters whether monopoly profits are distributed in such a way that they end up in households' liquid or illiquid accounts. In our preferred calibration, we find that amplification is stronger when a larger fraction of dividends are paid as liquid assets. Since profits are countercyclical in the model, the larger is the share of profits that are paid into the liquid account, and hence the smaller the share that is paid into the illiquid account, the smaller is the negative effect on illiquid balances in response to an expansionary shock. This means less drag on investment, which is paid from illiquid assets, and hence larger investment and a larger general equilibrium effect on household income.⁴ These findings further underscore the importance of how monopoly profits are distributed in HANK models. Broer, Harbo Hansen, Krusell, and Oberg (2016) have emphasized that, even in RANK models, the income effect of counter-cyclical profits on labor supply is crucial for the transmission of an interest rate cut. In HANK models, particularly those with both liquid and illiquid assets, the distribution of profits plays an even more critical

²The shape of the estimated incidence function with estimate SSA data is consistent with results in Guvenen, Schulhofer-Wohl, Song, and Yogo (2017), who use the same data set but with a slightly different specification and variable definitions.

 $^{^{3}}$ In the main text, we also explain that these findings are nonetheless consistent with Patterson (2018), who argues that her estimated incidence function results in amplification of up to 40 percent.

⁴There is also an offsetting effect, which we discuss further in the main text. The fall in profits that enter the liquid account dampen the aggregate aggregate consumption response. However, in our quantitative experiments, the investment effect dominates the consumption effect.

role (also see Werning, 2015).

Fourth, we find that the quantitative importance of the fiscal reaction function emphasized in Kaplan, Moll, and Violante (2018) is at least as strong as the other model elements that we consider, and is unaffected by the presence of capital adjustment costs. Whereas, because of Ricardian equivalence, the fiscal reaction function is inconsequential in RANK, assumptions about how the government balances its intertemporal budget constraint when its borrowing rate falls turn out to have a large effect in HANK because of the difference in MPCs between hand-to-mouth households, non hand-to-mouth households and the government. In our conclusions, we discuss some potential avenues for future work to investigate model features that drive this result, given its quantitative importance relative to other model elements that have received more attention.

XXX: DO WE NEED THIS PARA ON LITERATURE? HAPPY TO DROP IT OR WONE OF YOU TO CLEAN UP Our paper is, of course, part of the large and growing literature that integrates incomplete markets and nominal rigidities.⁵ Within this literature, the following strands are most closely related. First, as already mentioned, Auclert and Rognlie (2018) and Patterson (2018) emphasize the potential importance of unequal incidence of aggregate income dynamics on individual income as an amplification mechanism. The logic is simple: if high-MPC households are also those whose labor income is the most exposed to fluctuations in aggregate income, this will generate a larger impact of movements in aggregate income on aggregate consumption. In the context of a monetary shock, this implies larger general equilibrium effects and hence more amplification.

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The rest of the paper proceeds as follows. Section 2 describes how we estimate incidence functions. Section 3 outlines the model and calibration strategy. Section 4 collects the results from our experiments. Section 5 concludes the paper.

⁵See Guerrieri and Lorenzoni (2011), Oh and Reis (2012), Ravn and Sterk (2012), McKay and Reis (2016), Gornemann, Kuester, and Nakajima (2014), Auclert (2016), McKay, Nakamura, and Steinsson (2016), Den Haan, Rendahl, and Riegler (2015), Bayer, Luetticke, Pham-Dao, and Tjaden (2015), Luetticke (2015), and Werning (2015) XXX WE NEED TO UPDATE THE BIBTEX FILE XXX.

2 Incidence functions

An incidence function describes an allocation rule of an aggregate quantity across the distribution of households in the economy. It maps an aggregate variable and individual characteristics into individual-level values of that variable. To fix ideas, let y_{it} be the outcome variable of interest for individual *i* at time *t*, e.g. individual earnings, and let Y_t be its aggregate (or average) counterpart, $Y_t = \sum_i y_{it}$. Let z_{it} summarize a set of individual *i* characteristics, such as age, education, occupation, wealth, etc. The incidence function can be represented as $y_{it} = \Gamma(z_{it}, Y_t)$. The key parameters of this function that we will estimate are the elasticities $\gamma(z)$ which measure the individual exposure to fluctuations in the aggregate variables at different points of the distribution of *z*. XXX GK: NEED A FORMAL DEFINITION OF $\gamma(z)$. IN GENERAL IT DEPENDS ON *Y*: ARE WE SETTING *Y* TO SOME VALUE OR RESTRICTING ATTENTION TO FUNCTIONAL FORMS WHERE THE ELASTICITY DOES NOT DEPEND ON *Y*. WE NEED TO STATE THE FUNCTIONAL FORM XXX

In the rest of this section, we first explain why unequal incidence can affect the propagation mechanism of aggregate shocks, and then estimate incidence functions for labor earnings and government transfers from micro data.

2.1 Understanding role of incidence for amplification

It is useful to illustrate in more detail the mechanism by which unequal incidence may lead to amplification. To this end, we adopt the reduced form approach of Bilbiie (2017) and Patterson (2018) and consider the effect of changes in aggregate income on aggregate consumption in a simple static framework (essentially an adaptation of the classic Keynesian cross with heterogeneity).

There is a unit continuum of individuals indexed by *i*. Each individual's consumption c_i depends on her income y_i in a potentially non-linear fashion $c_i = g_i(y_i, \theta_i)$ where θ_i are other demand shifters. Aggregate consumption is $C = \mathbb{E}_i[c_i]$ and aggregate income is $Y = \mathbb{E}_i[y_i]$ where the expectation operator \mathbb{E}_i computes the cross-sectional average. Consider an aggregate shock, such as a monetary disturbance, that induces a change in aggregate income dY distributed across individuals in an unequal fashion dy_i but such that $\mathbb{E}_i[dy_i] = dY$. Denoting $MPC_i := \partial g_i/\partial y_i$, we can write the indirect general equilibrium effect of the shock on C, i.e. how the shock impacts aggregate consumption through the change in aggregate income as:

$$dC = \mathbb{E}_{i}[\mathrm{MPC}_{i} \cdot dy_{i}]$$

= $\mathbb{E}_{i}\left[\mathrm{MPC}_{i} \cdot \frac{y_{i}}{Y}\right] dY + \mathbb{COV}_{i}\left(\mathrm{MPC}_{i} \cdot \frac{y_{i}}{Y}, \gamma_{i}\right) dY$
= $\tilde{\mathbb{E}}_{i}[\mathrm{MPC}_{i}] dY + \mathbb{COV}_{i}(\mathrm{MPC}_{i}, \gamma_{i}) dY$

where $\gamma_i = \partial \log y_i / \partial \log Y$, \mathbb{COV}_i is the cross-sectional covariance and the $\tilde{}$ symbol denotes the income-weighted operator. The first term in the last line is simply the income-share weighted average MPC in the population times the change in total income dY. It shows that the size of the aggregate MPC of the economy affects the magnitude of the general equilibrium feedback of a shock. The second term is directly related to unequal incidence: it involves the covariance between individual income-weighted MPCs and the elasticity of individual income to aggregate income γ_i .

The covariance term is the term highlighted by Patterson (2018). If there is equal incidence, $\gamma_i = 1$ for all *i*, then the covariance term is zero. If individuals who are more exposed to fluctuations in aggregate income (high γ_i) are also those with high income-weighted MPCs, then this term is positive. In this case, unequal incidence is an amplification mechanism.

This simple exposition suggests that an analysis of the quantitative importance of unequal incidence as an amplification mechanism of monetary shocks requires two key ingredients. First, an empirically disciplined parameterization of the elasticities γ_i , which is what we discuss next. Second, a parameterization of how these elasticities co-vary with individuals' MPCs. Here, our approach differs from Patterson (2018). We do not make an attempt to estimate directly individual MPCs from micro data and correlate them with the degree of individual exposure to shocks, but we rely on the endogenous distribution of MPCs generated by our model. Section 3.2 articulates this point further.

2.2 Estimation from micro data

We now describe the estimation of the incidence functions for labor earnings and government transfers. We explained that the incidence function yields the elasticity of individual income to aggregate income as a function of individual characteristics.

It is known that individual traits such as gender, age, education, occupation, etc. are all determinants of the exposure to business cycles. For example, earnings and hours worked are more cyclical for women, younger workers, less skilled workers and for certain occupations and industries such as manufacturing. For ease of computation, we summarize all these characteristics into one variable only, the permanent component of labor income, which we denote by z.

We proceed in two steps. First, for each individual in the data we measure its position in the distribution of permanent income and bin individuals into quantiles. Next, we estimate a separate elasticity $\gamma(z)$ for each quantile. We start by estimating unconditional incidence functions, i.e. we include fluctuations in aggregate earnings caused all possible macro shocks. Next, we make an attempt to isolate only those caused by monetary shocks.

Our first data source is the Annual Social and Esonomic (ASEC) supplement of the Current Population Survey (CPS), which is conducted every March. This supplement to the CPS has the longest and largest sample as well as the most comprehensive collection of data on labor force status, work experience and different types of income. We use data from 1967 to 2017 for all individuals between the ages of 26 and 55. The total average annual sample size is around 66,000 observations per year.

Labor income is defined as total pre-tax wage and salary income –that is, money received as an employee– over the calendar year.⁶ Government transfers are defined as income received from Social Security, from all welfare programs (e.g., TANF, SNAPS, Housing Assistance), from other government programs other than Social Security and welfare (e.g., unemployment compensation, disability insurance), and from the Earned Income Tax Credit.

To construct our measure of permanent income, we first run a Mincer-style regression. We regress log labor income on dummies for gender, race, marital status, education, age and occupation, as well as interactions between education and age, and between gender and age, to capture some heterogeneity in life-cycle earnings profiles. The adjusted R^2 of these regressions varies between 0.32 and 0.47, with higher values in the earlier years. We then bin individuals into 50 quantiles based on their predicted level of permanent labor income.

Figure 1 plots the average labor and transfer income by quantile of permanent income z (left panel) and the share of the total accounted for by each quantile (right panel). As expected, labor income grows with z and especially so at the top, due to the skewness in the income distribution. The share of transfers is, instead, decreasing in permanent income with the low quantiles receiving five times the transfers of the highest earners. Recall that, in our discussion of the role of incidence for amplification of Section 2.1, we explained that what matters is the correlation between the elasticities and the MPCs weighted by the income shares.

Let $y_{it}(z)$ be labor income of individual *i* belonging to quantile *z* of permanent

 $^{^{6}}$ We also used a broader definition of labor income with an imputation of 2/3 of self-employment income and results are very similar. For both definitions, we dropped top-coded observations.



Figure 1: Levels and shares of labor income and government transfers by quantiles. The shares sum to one.

income in year t, and let Y_t be aggregate labor income. The final step to obtain the incidence function is to run the following regression separately for each quantile z:

$$\log y_{it}(z) = \beta_0(z) + \beta_1(z)t + \gamma(z)\log Y_t + \epsilon_{it}.$$
(1)

We add a linear trend to the regression because we are interested in the elasticity to *cyclical* movements in aggregate earnings. We allow the trend to be quantile-specific to capture the differential secular evolution of labor income at different points in the distribution related to the well document widening in U.S. earnings inequality. The coefficient of interest is the elasticity $\gamma(z)$ and, in particular, we want to document how it varies across the distribution of permanent income. Within each quantile of z, this coefficient is identified by the time variation in Y_t around the linear trend.

Figure 2 plots these elasticities across the distribution together with the 95 pct confidence interval. The left panel shows that the elasticity for low permanent income workers is 2-3 times larger than those of high permanent income ones.⁷

The log formulation in equation (1) presents a potential problem, however. Individual earnings are frequently zero in the data: more than 20% of all labor income observations in our sample are zeros and are concentrated in the lowest quantiles of of the distribution. For example, in the bottom decile, over half of the observations are zero, whereas at the top this fraction is less than 10%. When using the log to estimate the elasticity, these observations are dropped. Since the zeros are more likely to occur at times when aggregate earnings are low, one would expect this selection to

⁷Another strategy we adopted is to average earnings within a quantile and estimate the elasticity of mean earnings to aggregate earnings for each quantile. This regression gives results that are very similar to the regression in log.



Figure 2: Estimated elasticities of individual earnings to aggregate earnings as a function of permanent income quantile. Dotted lines are the 95% confidence bands. Source: ASEC 1967-2017.

produce a negative bias in $\gamma(z)$, especially at the low end of the permanent income distribution. To assess this bias, we replace the log operator in equation (1) with the inverse hyperbolic sine (asinh):

$$\operatorname{asinh}(y) = \log\left(y + \sqrt{y^2 + 1}\right).$$

This approach, which allows retaining zero-valued observations, is common in statistics (Bellemare and Wichman, 2018). The right panel of Figure 2 re-estimates equation (1) with this approach. The conclusions are quite stark. While above the 30th percentile the estimates of regression coefficient on aggregate earnings are roughly unchanged, at the bottom of the distribution the exposure to the cycle appears much stronger than in the left panel, and the hockey stick shape is much more pronounced.⁸

One caveat with these estimates is that we dropped top-coded observations from the sample. In addition, the CPS is known to undersample individuals at the very top of the earnings distribution. Therefore, these estimates do not reveal how sensitive the

$$\gamma\left(z|y_{it},Y_{t}\right) = \tilde{\gamma}\left(z\right) \cdot \sqrt{\frac{y_{it}^{2} + 1}{y_{it}}} \cdot \sqrt{\frac{Y_{t}}{Y_{t}^{2} + 1}}.$$

⁸There is the question of how to interpret the estimated coefficients from the regression that uses the asinh transformation. Let these coefficient be $\tilde{\gamma}(z)$ and the true elasticity be $\gamma(z)$. Bellemare and Wichman (2018) show that the elasticity is given by

Obviously, this elasticity cannot be computed at $y_{it} = 0$. Bellemare and Wichman (2018) suggest to evaluate it instead at the mean value for y_{it} . In practice, even for the lowest percentiles, the mean is large enough that $\gamma(z|y_{it}, Y_t) = \tilde{\gamma}(z)$.



Figure 3: Estimated elasticities of individual earnings to aggregate earnings as a function of permanent income quantile. Dotted lines are the 95% confidence bands. Source: Master Earnings File of the SSA 1979-2011.

very high-income households are to the cycle.

Guvenen, Schulhofer-Wohl, Song, and Yogo (2017) estimated "workers' betas" (i.e. systematic risk exposure) with respect to GDP using data from the Master earnings File of the Social Security Administration. These data are also annual, cover a shorter period of time (1981-2009) and have only information on earnings (not transfers), but have the key advantage of a much better coverage of the top end of the income distribution. Here, we use their same data but estimate incidence functions with respect to aggregate earnings, consistently with our framework.⁹

We report our findings in Figure 3. They key difference with Figure 2 is the fact that exposure increases significantly again for the very top earners, i.e. above the top 5-10 percent and markedly for the top 1 percent. A natural interpretation of these findings is that the high exposure at the bottom of the distribution is associated with unemployment risk, whereas at the top it is due to the fact that a large share of the compensation of high earners is made of performance-related bonuses and commissions.

Figure 4 repeats the exercise with government transfers in the ASEC sample. Also for this case, the log and asinh are quite similar in shape except at the lowest percentiles of permanent income. Once again, when measured with the asinh transformation, the exposure in the bottom decile appears much higher. Overall, the shape of the incidence function is not monotonic.

⁹Because of the format in which these data are available, the specification we use is not exactly the same as in (1). The measure of permanent income is the mean of the previous 5 years of earnings. Moreover, we estimate the equation in first differences, without any quantile-specific trend, i.e. the dependent variable is the log of the average change in earnings between t and t + 1 across all the individuals who were in quantile z at t.



Figure 4: Estimated elasticities of individual government transfer income to aggregate transfers as a function of permanent income quantile. Dotted lines are the 95% confidence bands. Source: ASEC 1967-2017.

3 The Model

Households Time is continuous. The economy is populated by a continuum of households who face an exogenous death rate η . Households receive a utility flow u from consuming $c_t \geq 0$ and inelastically supply one unit of labor. Actual hours worked are demand-determined, as we explain in more detail below. The function u is strictly increasing and strictly concave in consumption. Preferences are time-separable and, conditional on surviving, the future is discounted at rate $\rho \geq 0$:

$$\mathbb{E}_0 \int_0^\infty e^{-(\rho+\eta)t} u(c_t) dt,\tag{2}$$

where the expectation is taken over realizations of idiosyncratic labor productivity shocks. Individual labor productivity follows an exogenous stationary Markov process –which we describe in detail in Section 3.2– that is the product of a permanent component z and a transitory component ζ . Because of the law of large numbers, and the absence of aggregate shocks, there is no economy-wide uncertainty.

Households can hold non-negative positions in two types of real assets: a liquid asset b which pays a rate of return r_t^b , and an illiquid asset a. Assets of type a are illiquid in the sense that households need to pay a cost for depositing into or withdrawing from their illiquid account. Let d_t be a household's deposit rate (with $d_t < 0$ corresponding to withdrawals) and $\chi(d_t, a_t)$ be the flow cost of depositing at a rate d_t for a household with illiquid holdings a_t . As a consequence of this transaction cost, in equilibrium the illiquid asset pays a higher real return than the liquid asset, i.e. $r_t^a > r_t^b$.

Households are indexed by their holdings of liquid assets b, illiquid assets a, and by their idiosyncratic labor productivity pair (z, ζ) . At each instant in time t, the state of the economy is the joint distribution $\mu_t(da, db, dz, d\zeta)$. Upon death, households give birth to an offspring with zero wealth and labor productivity equal to a random draw from its ergodic distribution.¹⁰ There are perfect annuity markets so that the estates of the deceased are redistributed to other individuals in proportion to their asset holdings.¹¹

A household's asset holdings evolve according to

$$\dot{b}_{t} = (1 - \tau_{t})w_{t}\Gamma_{n}(z_{t}, \zeta_{t}, N_{t}) + r_{t}^{b}(b_{t})b_{t} + \Gamma_{T}(z_{t}, T_{t}) + \Gamma_{\pi}(z_{t}, \Pi_{t}) - d_{t} - \chi(d_{t}, a_{t}) - c_{t}$$
(3)

$$\dot{a}_t = r_t^a a_t + d_t \tag{4}$$

$$b_t \ge 0, \qquad a_t \ge 0. \tag{5}$$

Savings in liquid assets b_t equal the household's income stream (composed of labor earnings taxed at rate τ_t , interest payments on liquid assets, and government transfers) net of deposits into or withdrawals from the illiquid account d_t , transaction costs $\chi(d_t, a_t)$, and consumption expenditures c_t . Γ_n , Γ_T and Γ_{π} are incidence functions that capture how aggregate labor income, transfers and profits of intermediary producers are distributed across households. They depend on these aggregate quantities as well as on the idiosyncratic labor income states (z_t, ζ_t) .¹² In section 3.2, we describe these functions in more detail.

Net savings in illiquid assets \dot{a}_t equal interest payments on illiquid assets plus net deposits from the liquid account d_t . Note that while we distinguish between liquid and illiquid wealth, we net out gross positions within the two asset classes.

The functional form for the transaction cost $\chi(d, a)$ is given by

$$\chi(d,a) = \chi_1 \left| \frac{d}{a} \right|^{\chi_2} a. \tag{6}$$

The convexity $(\chi_1 > 0, \chi_2 > 1)$ ensures that deposit rates are finite, $|d_t| < \infty$ and hence

¹⁰We allow for stochastic death to help in generating a sufficient number of households with zero illiquid wealth relative to the data. This is not a technical assumption that is needed to guarantee the existence of a stationary distribution, which exists even in the case $\eta = 0$.

¹¹The assumption of perfect annuity markets is implemented by making the appropriate adjustment to the asset returns faced by surviving households. To ease notation, we fold this adjustment directly into the rates of return, which should therefore be interpreted as including the return from the annuity.

¹²More generally, the incidence functions could depend on the entire vector or individual states (a, b, z, ζ) or even the identity of each individual. Here we instead restrict it to depend only on the exogenous states, as we did for our empirical counterparts.

household's holdings of assets never jump. Finally, scaling the convex term by illiquid assets a delivers the desirable property that marginal costs $\chi_d(d, a)$ are homogeneous of degree zero in the deposit rate d/a so that the marginal cost of transacting depends on the fraction of illiquid assets transacted, rather than the raw size of the transaction.¹³

Households maximize (2) subject to (3)–(6). They take as given equilibrium paths for the real wage $\{w_t\}_{t\geq 0}$, the real return to liquid assets $\{r_t^b\}_{t\geq 0}$, the real return to illiquid assets $\{r_t^a\}_{t\geq 0}$, and taxes and transfers $\{\tau_t, \Gamma_T(\cdot, T_t)\}_{t\geq 0}$. As we explain below, $\{r_t^b\}_{t\geq 0}$ will be determined by monetary policy and a Fisher equation, and $\{w_t\}_{t\geq 0}$ and $\{r_t^a\}_{t\geq 0}$ will be determined by market clearing conditions for capital and labor.

Final-goods producers A competitive representative final-good producer aggregates a continuum of intermediate inputs indexed by $j \in [0, 1]$

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 0$ is the elasticity of substitution across goods. Cost minimization implies that demand for intermediate good j is

$$y_{j,t}(p_{j,t}) = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t$$
, where $P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$

Intermediate goods producers Each intermediate good j is produced by a monopolistically competitive producer using effective units of capital $k_{j,t}$ and effective units of labor $n_{j,t}$ according to the production function

$$y_{j,t} = k_{j,t}^{\alpha} n_{j,t}^{1-\alpha}.$$
 (7)

Intermediate producers rent capital at rate r_t^k in a competitive capital market and hire labor at wage w_t in a competitive labor market. Cost minimization implies that the marginal cost is common across all producers and given by

$$m_t = \left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha},\tag{8}$$

where factor prices equal their respective marginal revenue products.

¹³Because the transaction cost at a = 0 is infinite, in computations we replace the term a with max $\{a, \underline{a}\}$, where the threshold $\underline{a} > 0$ is a small value (always corresponding to \$500 in all calibrations) that guarantees costs remain finite even for households with a = 0.

Each intermediate producer chooses its price to maximize profits subject to price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic in the rate of price change \dot{p}_t/p_t and expressed as a fraction of aggregate output Y_t as

$$\Theta_t \left(\frac{\dot{p}_t}{p_t}\right) = \frac{\theta}{2} \left(\frac{\dot{p}_t}{p_t}\right)^2 Y_t,\tag{9}$$

where $\theta > 0$. Suppressing notational dependence on j, each intermediate producer chooses $\{p_t\}_{t\geq 0}$ to maximize

$$\int_0^\infty e^{-\int_0^t r_s^a ds} \left\{ \tilde{\Pi}_t(p_t) - \Theta_t\left(\frac{\dot{p}_t}{p_t}\right) \right\} dt,$$

where

$$\tilde{\Pi}_t(p_t) = \left(\frac{p_t}{P_t} - m_t\right) \left(\frac{p_t}{P_t}\right)^{-\varepsilon} Y_t \tag{10}$$

are flow profits before price adjustment costs. The choice of r_t^a for the rate at which firms discount future profits is justified by a no-arbitrage condition that we explain below.

As proved in Kaplan, Moll, and Violante (2018), the combination of a continuoustime formulation of the problem and quadratic price adjustment costs yields a simple equation (the New Keynesian Phillips curve) characterizing the evolution of inflation $\pi_t = \dot{P}_t/P_t$ without the need for log-linearization:

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\varepsilon}{\theta}\left(m_t - m^*\right) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon}.$$
(11)

Equation (11) can be also written in present-value form as:

$$\pi_t = \frac{\varepsilon}{\theta} \int_t^\infty e^{-\int_t^s r_\tau^a d\tau} \frac{Y_s}{Y_t} \left(m_s - m^* \right) ds.$$
(12)

The marginal gain to a firm from increasing its price at time s is $\Pi'_s(p_s) = \varepsilon Y_s (m_s - m^*)$. Firms raise prices when their markup $1/m_s$ is below the flexible price optimum $1/m^* = \frac{\varepsilon}{\varepsilon - 1}$. Inflation in (12) is the rate of price changes that equates the discounted sum of all future marginal payoffs from changing prices this period to its marginal cost $\theta \pi_t Y_t$ obtained from (9).

Investment Fund Illiquid assets are equity claims on an investment fund. Thus, the value of the fund equals households' aggregate stock of illiquid assets $A_t = \int a d\mu_t$.

The investment fund owns the economy's capital stock K_t and all the shares in the intermediate producers S_t . The fund makes the economy's investment decision subject to an adjustment cost $\Phi(\iota_t)$ where ι_t is the investment rate, i.e. investment as a fraction of the capital stock. The shares S_t represent a claim on a fraction α of the entire future stream of monopoly profits net of price adjustment costs, $\Pi_t := \tilde{\Pi}_t - \frac{\theta}{2}\pi_t^2 Y_t$. Let q_t^s denote the share price. The remaining fraction $1 - \alpha$ of profits flows directly into households' liquid asset account as we explained above.

The investment fund chooses $\{\iota_t, S_t\}_{t\geq 0}$ to maximize

$$\int_0^\infty e^{-\int_0^t r_s^a ds} \left\{ [r_t^k - \iota_t - \Phi(\iota_t)] K_t + \alpha \Pi_t S_t - q_t^s \dot{S}_t \right\} dt$$

subject to

$$K_t = (\iota_t - \delta) K_t,$$

with K_0 and S_0 given.

Lemma 1 The optimal investment rate ι_t satisfies $1+\Phi'(\iota_t) = q_t^k$ where $q_t^k := \partial A(K_t, S_t)/\partial K_t$ is the fund's shadow value of capital. The value of the fund is given by $q_t^k K_t + q_t^s S_t$. And the return to illiquid assets r_t^a satisfies

$$r_t^a = \frac{r_t^k - \iota_t - \Phi(\iota_t) + q_t^k(\iota_t - \delta) + \dot{q}_t^k}{q_t^k} = \frac{\alpha \Pi_t + \dot{q}_t^s}{q_t^s}.$$
 (13)

Note that the arbitrage condition (13) pins down the return on the illiquid asset r_t^a . Finally, note that (13) implies that $q_t^s = \alpha \int_t^\infty e^{-\int_t^\tau r_s^a ds} \prod_\tau d\tau$ which justifies the use of r_t^a as the rate at which future profits are discounted by the intermediate firms and, thus, as the discount rate appearing in equation (11), the Phillips curve.

Labor Market Our modeling of the labor market is non-standard. As already mentioned, we assume that aggregate employment and wages are determined from firms' labor demand together with an exogenous wage-setting rule. The labor demand schedule comes from intermediate firms' profit maximization and pins down employment N_t as a function of wages w_t and a number of demand-shifters. To determine wages we assume an exogenous wage-setting rule

$$w_t = \bar{w} \left(\frac{N_t}{\bar{N}}\right)^{\epsilon_w},\tag{14}$$

where N_t is aggregate employment and \bar{w} and \bar{N} are steady state wages and employment. For instance, if $\epsilon_w = 0$, wages are perfectly rigid and employment is simply determined by the location of firms' labor demand schedule. If $\epsilon_w > 0$, there is downward pressure on wages whenever employment is below its steady state value. Aggregate employment is then distributed across households according to the incidence function $\Gamma_n(z_t, \zeta_t, N_t)$.

In Kaplan, Moll, and Violante (2018) we adopted the assumption of the basic New Keynesian model that prices are sticky while wages are flexible. As a result, markups are countercyclical under a monetary shock. In practice, this typically also implies that profits decrease sharply after a monetary expansion. It is by now well understood in the HANK literature that the distribution of profits can have large effects on the model's cyclical properties of aggregate consumption and output.(e.g. Werning, 2015; Broer, Harbo Hansen, Krusell, and Oberg, 2016) Falling profits in response to expansionary monetary shocks is counterfactual. The advantage of our assumption is that we can control the degree of wage rigidity in the economy. When wages are rigid, intermediate firms' marginal costs and markups move less in response to shocks. Therefore the dynamics of profits, dividends and equity prices are less counterfactual.

Monetary Authority The monetary authority sets the nominal interest rate on liquid assets i_t according to a Taylor rule

$$i_t = \bar{r}^b + \phi \pi_t + \epsilon_t \tag{15}$$

where $\phi > 1$ and $\epsilon_t = 0$ in steady state. Our main experiment studies the economy's adjustment after an unexpected temporary monetary shock ϵ_t .

Given inflation and the nominal interest rate, the real return on the liquid asset is determined by the Fisher equation $r_t^b = i_t - \pi_t$. The real liquid return r_t^b needs also to be consistent with equilibrium in the bond market, which we describe in Section 3.1.

Government The government faces exogenous government expenditures G_t and administers a progressive tax and transfer scheme on household labor income that consists of a lump-sum transfer T_t and a proportional tax rate τ_t . The government is the sole issuer of liquid assets in the economy, which are real bonds of infinitesimal maturity B_t^g , with negative values denoting government debt. Its intertemporal budget constraint is

$$\dot{B}_t^g + G_t + T_t = \tau_t w_t N_t + r_t^b B_t^g \tag{16}$$

Outside of steady state, the fiscal instrument that adjusts to balance the budget can be either τ_t , T_t , or G_t . In our experiments, we consider various alternatives.

3.1 Equilibrium

An equilibrium in this economy is defined as paths for individual household and firm decisions $\{a_t, b_t, c_t, d_t, n_t, k_t\}_{t\geq 0}$, input prices $\{w_t, r_t^k\}_{t\geq 0}$, returns on liquid and illiquid assets, $\{r_t^b, r_t^a\}_{t\geq 0}$, the share price $\{q_t^s\}_{t\geq 0}$, the inflation rate $\{\pi_t\}_{t\geq 0}$, fiscal variables $\{\tau_t, T_t, G_t, B_t\}_{t\geq 0}$, distributions $\{\mu_t\}_{t\geq 0}$, and aggregate quantities such that, at every t: (i) households and firms maximize their objective functions taking as given equilibrium prices, taxes, and transfers; (ii) the sequence of distributions satisfies aggregate consistency conditions; (iii) the government budget constraint holds; and (iv) the liquid asset (bond) market, markets for capital and shares of the intermediate firms (that can be folded into a single illiquid asset), and the goods market all clear.

The liquid asset market clears when

$$B_t^h + B_t^g = 0, (17)$$

where B_t^g is the stock of outstanding government debt and $B_t^h = \int b d\mu_t$ are total household holdings of liquid bonds. In equilibrium the investment fund holds all the shares in intermediary producers which we normalize to one so that $S_t = 1$. From Lemma 1 this implies that households' holdings of illiquid assets $A_t = \int a d\mu_t$ equals

$$A_t = q_t^k K_t + q_t^s, (18)$$

The goods market clearing condition is:

$$Y_t = C_t + I_t + G_t + \Theta_t + \Phi_t + \chi_t.$$

$$\tag{19}$$

Here, Y_t is aggregate output, C_t is total consumption expenditures, I_t is gross additions to the capital stock K_t , G_t is government spending, Θ_t and Φ_t are total price and capital adjustment costs, and the last two term reflects transaction costs (to be interpreted as financial services).

As explained, the labor market does not clear. The aggregation of intermediate producers' labor demand determines N_t and, given N_t , equation (14) determines the wage.

3.2 Calibration

Demographics and Preferences We set the quarterly death rate to 1/180 so that the average lifespan is of 45 years. Households have CRRA utility over consumption with risk aversion parameter γ set to 1.

Our calibration is divided into three main steps. First, we calibrate the exogenous stochastic process for idiosyncratic labor productivity. Second, we target a realistic distribution of liquid and illiquid assets and the fraction of households with low liquid wealth as this directly maps to the distribution of MPCs, which is key to consumption response as highlighted in section 2.1. Finally, we calibrate parameters of the production and monetary side of the model to standard values of the New Keynesian literature. The list of parameter values is in Table 2.

Continuous Time Earnings We take the processes for individual labor productivity (z_{it}, ζ_{it}) from Kaplan, Moll, and Violante (2018). Each component is modeled as a "jump-drift" process in logarithms. Let the logarithm of the permanent component be $\tilde{z}_{it} \equiv \log z_{it}$. Jumps arrive at some Poisson intensity λ_z and upon their realization a new value for the state \tilde{z}'_{it} is drawn from a normal distribution with mean zero and variance σ_z^2 , $\tilde{z}'_{it} \sim \mathcal{N}(0, \sigma_z^2)$. Between jumps, the process simply reverts to zero at some rate β_z . Formally, the process for \tilde{z}_{it} is

$$d\tilde{z}_{it} = -\beta_z \tilde{z}_{it} dt + dJ_{z,it}$$

where $dJ_{z,it}$ captures the jumps in the process. The description of the transitory component is analogous. The process is estimated to replicate the higher-order moments of the distribution of earnings changes estimated by Guvenen, Karahan, Ozkan, and Song (2015) from SSA data. The Poisson shock of the permanent component z occurs on average once every 38 years and the process has a half-life of around 18 years. The transitory component ζ jumps on average once every 3 years and the process has a half-life of around one quarter.¹⁴

Wealth Distribution We set steady-state nominal return on liquid asset at 2 percent per year and inflation at zero. The steady state return on the illiquid asset is endogenous.

Taking as given the process for idiosyncratic labor productivity, households' incentives to accumulate liquid and illiquid assets depend mainly on the discount rate ρ and the parameters of the transaction cost function χ_1, χ_2 (recall that we have assumed away unsecured borrowing). We choose these parameters to match four moments of the household wealth distribution: (i)-(ii) mean of liquid and illiquid wealth over annual

¹⁴See Table 3 in Kaplan, Moll, and Violante (2018) for the fit and the exact parameter values. Overall, the fitted earnings process matches the variance and kurtosis of 1 and 5 year earnings changes, as well as fraction of small changes. Consistent with cross sectional earnings distribution in the data, our earnings process features a large amount of right-tail inequality. The top 10, 1 and 0.1 shares of gross household labor earnings in the steady state are 46, 14 and 4 percent respectively.

	Data	Model
Mean illiquid asset	2.92	2.88
Mean liquid asset	0.20	0.21
Frac. with $b \approx 0$ and $a = 0$	0.10	0.06
Frac. with $b \approx 0$ and $a > 0$	0.20	0.25

Notes: Approximately 0 stands for $b \in [0, \underline{b}]$ where we set \underline{b} to 5 per cent of quarterly labor income or around \$800.

Table 1: Targeted empirical moments for the wealth distribution (ratios of net asset positions to annual GDP) and the share of hand to mouth households (relative to the total population), with their model counterpart.



Figure 5: caption

GDP from Kaplan, Moll, and Violante (2018), (iii)-(iv) fraction of poor and wealthy hand-to-mouth households.¹⁵ Table 1 shows the fit of the model with respect to these targets. The implied steady-state return on illiquid assets r^a is 6.6 percent per annum.

Figure 5 displays the steady state distributions of liquid and illiquid wealth for this calibration. The Gini coefficients in the model are 0.75 and 0.79 for the liquid and illiquid wealth distributions respectively, which imply a Gini coefficient for net worth very close to its empirical counterpart of 0.81.¹⁶

 $^{^{15}}$ The definition of poor and wealthy hand-to-mouth follows the one adopted by Kaplan and Violante (2014) and Kaplan, Violante, and Weidner (2014), i.e. it is based on the ratio between liquid wealth holdings and income.

¹⁶In the resulting ergodic distribution, roughly 85 percent of households are adjusting at any point in time. Conditional on making a deposit or withdrawal, the mean absolute quarterly transaction as a fraction of the stock of illiquid assets is 2.3 percent. The transaction cost associated with a transaction this size is 11 percent of the transaction. In steady state, the equilibrium aggregate transaction costs, which one can interpret as financial services, amount to less than 3 percent of GDP.

Production and Labor Market The elasticity of substitution for final goods producers ε is set to 10. In the production function of intermediate goods producers we set to $\alpha = 0.33$, which yields a capital share of 29% and labor share of 60%. The price adjustment cost parameter θ is set to 100 so that the slope of the Phillips curve ε/θ is 0.10.

When we solve models with the capital adjustment cost, we adopt the following specification for the function $\Phi(\cdot)$:

$$\Phi(\iota) = \frac{\phi_0}{2} \left(\iota - \delta\right)^2 \tag{20}$$

where δ is the depreciation rate. We set ϕ_0 to 25 in order so that when the economy is hit by a monetary shock, at its peak, the ratio of investment to output is around 2, in line with VAR evidence presented by Christiano, Eichenbaum, and Trabandt (2015).

The wage elasticity to aggregate hours ϵ_w in the wage setting rule is set to 0.10.¹⁷

Fiscal and Monetary Policy We set the proportional labor income tax rate τ to 0.30 and the lump-sum transfer T to be 2.3% of output. Since the government is the only provider of liquid assets, government debt is 21% of annual GDP –the target in Table 1. Government expenditures are determined residually from the government budget constraint. The Taylor rule coefficient ϕ is set to 1.25 which is in the range of commonly used for New Keynesian models.

Distribution of Monopoly Profits In our two-asset model, we need to take a stand on whether profits paid out as dividends end up in a household's liquid or illiquid accounts. This matters because the MPC out liquid resources is much larger than the MPC out of illiquid resources, due to the transaction cost.

In our baseline, we assume that monopolistic profits Π_t are split between dividends paid to the illiquid investment fund and dividends paid directly into liquid accounts in proportion to $(\alpha, 1 - \alpha)$, respectively. As discussed in Kaplan, Moll, and Violante (2018), this particular choice "neutralizes" the distributional consequences (with respect to aggregate liquidity) of countercyclical profits. The profits received by the investment fund end up in illiquid wealth and their distribution across individuals is pinned down by households endogenous accumulation of illiquid assets. Profits flowing into the liquid account are distributed across households through Γ_{π} in proportion in-

 $^{^{17}}$ Taking into account the confidence intervals in Figure 1 of Christiano, Eichenbaum, and Trabandt (2015), the elasticity of wage to hours in response to a monetary shock can be placed between anywhere between 0.0 and 1.00. As explained, we choose a value closer to the lower bound to reduce the movement in marginal cost, and hence the movement in profits.



Figure 6: Approximation of the estimated unequal incidence functions to be used in simulation

dividuals' labor income, i.e. $\Gamma_{\pi}(z_{it}, \zeta_{it}, \Pi_t) = z_{it}\zeta_{it}(1-\alpha)\Pi_t$. This specific distribution rule reflects the fact that a sizable share of labor compensation is in terms of bonuses and commissions linked to firm's performance.

Incidence Functions Our specification of the incidence functions Γ_n and Γ_T that enter households' budget constraint (3) is such that the fraction of aggregates allocated across households varies over the business cycle in a parsimonious fashion. Using a recursive formulation to ease notation, we assume that

$$\Gamma_n(z,\zeta,N) = \frac{z\zeta(N/\bar{N})^{\gamma_n(z)}}{\int z'\zeta'(N/\bar{N})^{\gamma_n(z')}d\mu_t}N,$$

$$\Gamma_T(z,T) = \frac{\nu(z)(T/\bar{T})^{\gamma_T(z)}}{\nu(z')\int (T/\bar{T})^{\gamma_T(z')}d\mu_t}T,$$

where \bar{N} and \bar{T} are steady state aggregates, and $\nu(z)$ is the steady-state share of transfers accruing to type z, calibrated based on Figure 1, right panel. The functions γ_n, γ_T satisfy the normalization conditions $\int z \zeta \gamma_n(z) d\bar{\mu} = \int \nu(z) \gamma_T(z) d\bar{\mu} = 1$ where $\bar{\mu}$ denotes the stationary distribution.¹⁸

Under this assumption, the elasticities of individual employment and transfers to

¹⁸The model reproduces very precisely the share of labor income and transfers by permanent income in steady-state by construction. The share of transfers at each level of z can be generated exactly in calibration. The share of labor earnings is not exact because our process for individual labor earnings is estimated with data from the Master Earnings File of the Social Security Administration, rather than CPS. However, the correspondence is very close.

Description		Value	Target	
Preferences				
η	Death rate	1/180	Avg. lifespan of 45 years	
$1/\gamma$	Y Intertemporal elasticity of substitution	1.00		
ρ	Discount rate (p.a.)	7.2%	See Table 1	
Tre	ansaction cost function			
<u>a</u>	Min a in denominator	\$500.00	See Table 1	
χ_1	Level component	0.395	See Table 1	
χ_2	Convex component	1.326	See Table 1	
Pre	pduction			
ε	Demand elasticity	10	Profit share of 10%	
θ	Price adjustment cost	100	Slope of Phillips cuve $\epsilon/\theta = 0.1$	
α	Capital share	0.33	National Accounts	
δ	Depreciation rate (p.a.)	5.75%	National Accounts	
ϕ_0	Capital adj. cost	[0, 25]	VAR evidence	
Labor Market				
ϵ_w	Wage elasticity	0.10	VAR evidence	
Go	vernment			
au	Proportional labor tax	0.30	National Accounts	
T	Lump-sum transfer (rel GDP)	0.027	Transfer GDP share of 3%	
ϕ_{π}	Taylor rule coefficient to inflation	1.25		

Table 2: List of parameter values and targeted moments



Figure 7: Share of hand-to-mouth households in the data and in the model

their aggregate counterparts are:¹⁹

$$\frac{\partial \log \Gamma_n(z,\zeta,\bar{N})}{\partial \log N} = \gamma_n(z), \qquad \frac{\partial \log \Gamma_T(z,\bar{T})}{\partial \log T} = \gamma_T(z).$$

The left panel of Figure 6 plots the incidence functions for earnings that we use in our experiments. Equal refers to the neutral baseline where each individual has equal exposure to shocks. GSWSY approximates the incidence function estimated on the SSA data following Guvenen, Schulhofer-Wohl, Song, and Yogo (2017). Extreme approximates our estimated incidence function using the asinh transformation on ASEC data. US is an average between the last two. The right panel of Figure 6 plots the incidence function for government transfers that we use in our experiments which approximates the right panel of Figure 4.

Recall from section 2.1 that what matters for amplification through unequal incidence is the covariance between MPC and the elasticity across the z distribution. As discussed earlier, one difference with Patterson (2018) is that, instead of estimating the distribution of MPCs out of unexpected transitory income changes —an arduous empirical task— we rely on the model. Therefore, we want to be confident that our distribution of MPCs (not just its first moment) is in line with the data. We exploit

$$\log \Gamma_n(z,\zeta,N) = \log(z\zeta) + \gamma_n(z) \log \left(N/\bar{N}\right) + \log N - \log \int z'\zeta' e^{\gamma_n(z) \log(N/\bar{N})} d\mu.$$

¹⁹Consider for example the labor incidence function Γ_n . We have

Differentiating with respect to $\log N$, evaluating at $N = \overline{N}$ and using that $\int z\zeta d\mu = 1$ and $\int z\zeta \gamma_n(z)d\mu = 1$, we have $\partial \log \Gamma_n(z,\zeta,\overline{N})/\partial \log N = \gamma_n(z)$. The derivations for the elasticities of Γ_T is similar.



Figure 8: Quarterly MPC out of additional \$500 of liquid and illiquid wealth as a function of liquid holdings.

the result that in our model there is a very close correspondence between share of HtM and level of the MPC, something we showed already in Kaplan, Moll, and Violante (2018). This result is useful because hand-to-mouth households are observable in the micro data.

Figure 7 plots the share of hand to mouth (HtM) households in the data and in the model for each quantile of permanent income. Not surprisingly, the share of HtM households is declining in permanent income in the data (left panel).²⁰ The model replicates this empirical pattern quite well.²¹ This match gives us some confidence that the distribution of MPC by z that emerges endogenously from the data is empirically plausible.

Figure 8 plots the distributions of MPCs out of a small increase in liquid and illiquid wealth. Note that the MPC out of illiquid wealth is simply the MPC out of liquid wealth times the share of the wealth increase withdrawn, net of the transaction cost. It is fairly stable around the distribution, averaging 3% quarterly (5 times smaller than the MPC out of transitory income).

3.3 Baseline results on monetary transmission

We simulate the transitional dynamics of the model economy in response to a one-time unexpected expansionary monetary shock. We always consider the same experiment: at time t = 0, there is a quarterly innovation to the Taylor rule (15) of $\epsilon_0 = -0.25\%$

 $^{^{20}}$ The data source we used for these calculations is the Survey of Consumer Finances 1989-2016. The sample selection is the same as in the CPS, and so is the Mincer regression to impute permanent income.

 $^{^{21}}$ The large flat region corresponds to the mid point in the permanent income distribution which, in the discretization, has a large share of the total mass — a consequence of its kurtosis.



Figure 9: Aggregate responses to a monetary policy shock. Top panels: quantities. Bottom panels: equity price.

(i.e. -1% annually) that mean-reverts at rate η , i.e. $\epsilon_t = e^{-\eta t} \epsilon_0$. We set $\eta = 0.5$, corresponding to a quarterly autocorrelation of $e^{-\eta} = 0.61$, a value consistent with the VAR-based empirical evidence.

In our baseline we make the following four assumptions that we will relax in the experiments of the next section: (i) equal incidence; (ii) a fraction α (the capital share) of profits is paid out in liquid form proportionately to individuals' labor income; (iii) the government budget constraint adjusts via deficit and surpluses in the short run, and in the longer run transfers adjust to get the level of debt back to its steady-state value; (iv) the Taylor rule specifies that the monetary authority does not respond to output, but only to inflation.

Figure 9 plots the impulse response functions (IRFs) for output, consumption, investment and the equity value of the fund in the baseline for the model without and with capital adjustment costs. Without adjustment costs, aggregate investment responds more to the shock, but the equity price barely moves: if anything, it falls slightly at impact because profits of the intermediaries decline after a monetary expansion. With adjustment costs, the investment response is very weak and the increase in demand for capital is absorbed by prices. This result highlights a shortcoming of this



Figure 10: Decomposition of the consumption IRF to a monetary shock into direct and indirect components.

first generation of HANK models: in the data, after monetary shocks both investment and asset prices respond rather strongly. In contrast to output, the IRF for aggregate consumption is unaffected by the presence of adjustment costs.

Figure 10 decomposes the IRF for aggregate consumption into direct and indirect effects, following Kaplan, Moll, and Violante (2018). Direct and indirect effects are computed by counterfactuals. To compute the direct impact of a monetary shock, we let the real liquid rate change as in the baseline, but freeze all other prices and government transfers at their steady-state value. Indirect effects are computed in a similar way, for each price. The figure splits indirect effects between the impact on consumption caused by the change in disposable labor income and the one due to the change in the equity value.

Both with and without adjustment costs, the indirect general equilibrium channel accounts for about half of the total increase in aggregate consumption at impact, but virtually all of it after a year or so already. This stark difference from the representative agent version of the New Keynesian model —where intertemporal substitution dominates the transmission mechanism at all frequencies— is the main conclusion of Kaplan, Moll, and Violante (2018).

The initial impulse to consumption always stems from the direct channel, i.e. the aggregation across non hand-to-mouth households of substitution and income effects of the interest rate cut. The propagation of this initial impulse, however, differs across the two cases. Without adjustment costs, consumption increases mostly because of higher labor income. The increase in demand for final goods pushes up the demand for capital and, as a result, employment and labor income. Hand-to-mouth-households with high



Figure 11: Decomposition of the impact effect of a monetary shock on consumption across the distribution of liquid wealth.

MPCs respond strongly to the rise in income, and hence expenditures increase. With adjustment cost, the spike in labor demand, and the implied rise in labor income, are more muted. However, equity prices increase and many households consume a share of this capital gain (recall Figure 8). Thus, we conclude that the transmission channel of monetary policy is quite different in these two cases.

Figure 11 illustrates that the consequence of this different transmission mechanism is that who responds to/gains from the monetary expansion also differs. Without capital adjustment costs, poor households benefit the most because of the larger role of labor income. With high capital adjustment costs, wealthier households benefit more because of the larger role of equity prices.

With these baseline results in hand, we now turn to the question of how the addition of our various potential amplification mechanisms affects the monetary transmission mechanism and to what extent it amplifies the consumption response to a monetary expansion.

4 What matters for amplification? Quantitative experiments

We consider the same one-time unexpected monetary shock as in Section 3.3. But now we "switch on" various potential amplification sources one by one so as to gauge their quantitative importance.

4.1 Unequal income incidence

We first use our model to ask to what extent unequal income incidence can serve as an amplification mechanism, as suggested by Bilbiie (2017) and Patterson (2018). Figure 12 plots the impulse response of aggregate consumption to the same monetary



Figure 12: Consumption response to monetary shock with unequal labor incidence

expansion as in Figure 9 under different parameterizations for the labor incidence function Γ_n .

Comparing the consumption response with our preferred estimate for incidence to that with equal incidence (i.e., comparing the lines labelled "US" and "equal"), we see that unequal incidence generates a small amount of amplification relative to the equal incidence case. For example, over the first quarter, unequal incidence increases the aggregate consumption response at impact from 0.33% to 0.36% percentage points which is an amplification of 10%. After one year, this amplification is barely noticeable, and is even smaller at all horizons in the presence of capital adjustment costs.

Perhaps surprisingly, the GSWSY calibration of the incidence function yields no amplification, relative to the equal incidence case (and actually yields a very small dampening). To understand why, recall that for the GSWSY incidence function, the elasticity of individual hours to aggregate hours is U-shaped, meaning that incomes at both the bottom and the top of the distribution are more exposed to fluctuations in aggregate incomes than those in the middle. There are therefore two offsetting forces at work. More exposure at the bottom, where MPCs are higher than average, leads to amplification; but more exposure at the top, where MPCs are lower than average, leads to dampening. Furthermore, recall from Section 2.1 that it is the *incomeweighted* covariance between MPCs and the elasticities, \mathbb{COV}_i (MPC_i, γ_i), that matters for amplification. Since individuals at the top of the distribution receive a higher share of aggregate income, the upward-sloping part of the GSWSY incidence function receives higher weight than the downward-sloping part. The net effect is that the GSWSY incidence function yields a slightly smaller consumption response than our baseline with equal incidence. Is it always true that unequal incidence yields only modest deviations (either positive or negative) from our baseline results? Our final experiment, based on the "extreme" incidence function suggests the answer is "no". With this parameterization, unequal incidence increases the aggregate consumption response on impact from 0.33% to 0.44% percentage points which is a sizable amplification of over 30%. But even with this incidence function, sizable amplification is achieved only in the absence of capital adjustment costs, and is relatively short-lived. Unequal income incidence can in principle yield large amplification if one takes the view that the data lie at the high end of the range of estimates we provided.

Finally, it is useful to explain how our results fit in with work by Patterson (2018) who argues that, under her estimated unequal income incidence function, monetary shocks are amplified by up to 40 percent over a benchmark in which all workers are equally exposed. In fact, our conclusions are entirely consistent with hers. The difference is due to how the respective findings are reported. In Patterson's baseline results, the general equilibrium multiplier increases from 1.3 to 1.42 (see her Table 3). Depending on one's viewpoint, this finding can be interpreted as an increase of 40 percent in the multiplier (from 0.3 to 0.42, as Patterson does) or an increase of 9 percent in the deviations of aggregate consumption from steady state $(1.42/1.3, as we did in the preceding discussion).^{22}$

4.2 Profit distribution

Our next candidate for the amplification of monetary shocks is the distribution of profits outside of steady state. Recall that in our baseline we assumed that (i) a fraction $1 - \alpha$ (the labor share) of profits was paid out as liquid lump-sum transfers and (ii) that they were paid out proportionately to individuals' labor income $z_{it}\zeta_{it}$. As discussed in Kaplan, Moll, and Violante (2018), these assumptions aim to "neutralize" as far as possible the distributional consequences of countercyclical profits.

We now generalize this profit distribution as follows. We first focus on "profit distribution across assets". To this end, we work with the following generalized function

²²Formally, returning to simple model of Section 2.1, the total effect of a monetary shock on aggregate consumption can be written as $dC = \mathbb{M}\tilde{\mathbb{P}}\mathbb{C}dY + \mathbb{C}\tilde{\mathbb{O}}\mathbb{V}dY + \mathbb{D}dr$, where the first two terms represent the general equilibrium component and $\mathbb{D}dr$ represents the direct effect of the shock at impact. Using the equilibrium condition C = Y, it is immediate that the total effect can be written as $dC/dr = 1/(1 - \mathbb{M}\tilde{\mathbb{P}}\mathbb{C} - \mathbb{C}\tilde{\mathbb{O}}\mathbb{V})\mathbb{D}$. In Patterson (2018), the GE multiplier with equal incidence (with $\mathbb{C}\tilde{\mathbb{O}}\mathbb{V} = 0$) is estimated to be 1.3 and with unequal incidence 1.42. Therefore, adding unequal incidence amplifies the rise of C at impact by $(1.42/1.3 - 1 \times 100)$ percent, i.e. 9 percent.



Figure 13: Consumption response to monetary shock for different profit distributions. Left panel: different shares going to liquid and illiquid account. Right panel: all dividends going to the liquid account, bu distributed differently across households.

for profit distributions into the liquid account

$$z_{it}\zeta_{it}\left[(1-\alpha)\bar{\Pi}+(1-\omega)(\Pi_t-\bar{\Pi})\right],$$

where Π denotes steady-state monopoly profits and $(1 - \omega)$ denotes the deviations of profits from steady state that are paid out as liquid dividends. The remaining profits are always received by the investment fund, that is, they end up in illiquid wealth and their distribution across individuals is pinned down by households' accumulation of illiquid assets (which are held in the investment fund).

We consider different scenarios corresponding to three different values of ω . First, our baseline model corresponds to the case $\omega = \alpha$ so that the same fraction of profits $1 - \alpha$ ends up in liquid wealth, both in and out of steady state. Second, we consider the case $\omega = 0$ meaning that all profit deviations from steady state are paid out in liquid form. Third, with $\omega = 1$ all profit deviations end up in illiquid wealth. Finally, to gauge the importance of distribution across individuals, we consider a fourth scenario in which all profits are paid out as liquid wealth but they are equally distributed across individuals rather than proportionately to their labor income as in our baseline results.

Figure 13 summarizes the results for the economy without capital adjustment costs. The left panel focuses on the first three alternative profit distribution rules based on the different values for ω . When a high share of countercyclical profits is paid into the liquid account (low ω), it curtails the expansion of household disposable income

and consumption. When it is paid into the liquid account (high ω), it drags down the investment expansion resulting from the monetary shock. Less investment implies lower demand for labor and lower household disposable income. In our experiment, it is the second of these two offsetting forces that dominates and aggregate consumption increases more the larger is the fraction of profits paid as liquid assets, thanks to the expansionary effect on investment. Quantitatively, these differences are larger than the differences induced by different labor incidence functions.

Next, we turn to "profit distribution across individuals" in the right panel. The dashed line labelled "all into liquid, equally distributed" plots results for our scenario in which deviations of profits from steady state are equally distributed across individuals rather than proportionately to their labor income. In this case, consumption responds less to a monetary expansion. Poor households are now more exposed to the countercyclical profits and, since those households have high MPCs, this exposure drags down the aggregate consumption response. With capital adjustment costs, our findings are qualitatively unchanged, but the different profit distribution rules generate more smaller differences in amplification.

4.3 Fiscal adjustment to the monetary shock

We next examine how important the fiscal reaction to the monetary expansion is for determining the size of the aggregate consumption response. Kaplan, Moll, and Violante (2018) have argued that in HANK the consequences of monetary policy are intertwined with the fiscal side of the economy, because of the failure of Ricardian equivalence. Since the government is a major issuer of liquid obligations, a change in the interest rate necessarily affects the intertemporal government budget constraint and generates some form of fiscal response that affects household disposable income – see equation (16).

In our baseline results, we assumed that government debt absorbs the majority of the fiscal imbalance in the short run. We consider two alternatives to this scenario that differ according to the fiscal instrument that adjusts to make the government budget constraint hold (expenditures and transfers). For the case where transfers adjust, we analyze whether the unequal transfer incidence function has an effect.

The left panel of Figure 14 summarizes the results for the two alternative fiscal adjustments together with the baseline (debt adjusts case). In the first scenario, government expenditures and debt are held constant, and transfers adjust. With transfers adjusting, amplification of aggregate consumption is larger compared to the baselines. This finding was already emphasized by Kaplan, Moll, and Violante (2018) and the intuition is simple: when interest rates fall, the government pays less interest on its debt



Figure 14: Consumption response to monetary shock for different fiscal adjustments. Left panel:

and this frees up additional resources that are "handed out" as transfers; households' disposable income rises and generates an additional impulse to aggregate consumption.²³

In the alternative scenario, government expenditures adjust at every instant so that the government budget constraint holds while transfers and government debt are held at their steady state levels. This scenario again leads to more amplification than the baseline case with debt adjusting. The intuition is similar: when government expenditures adjust, the reduced interest payments on debt translate one-for-one into an increase in aggregate demand, which contributes directly to an increase in household incomes and hence consumption. However, the amplification is smaller than when transfer adjust because there are only indirect general equilibrium effects, but no direct impact on households budget constraint.

Figure 14 also illustrates how different assumptions about how the government budget constraint adjust generate much larger differences in the aggregate consumption response than do different assumptions about labor income incidence. Unlike in the discussion are profits in the previous section, the presence or absence of capital adjustment costs has almost no impact on the size of these differences in the aggregate consumption response.

The right panel of this figure shows that the two "transfers adjust" scenarios with equal and with unequal incidence are very close to each other. In the experiment

 $^{^{23}}$ In contrast to the present paper, adjusting transfers was the baseline scenario in Kaplan, Moll, and Violante (2018). That paper did not consider unequal transfer incidence and everyone received the same transfers, both in and out of steady state.



Figure 15: Decomposition of the IRF for consumption. Left panel: Taylor rule without output gap. Right panel: Taylor rule with output gap.

corresponding to the line labelled "transfers adjust, equal incidence", transfers adjust proportionately for all households, that is, the elasticity of the transfer incidence function is $\gamma_T(z) = 1$ for all z. In the experiment corresponding to the line labelled "transfers adjust, US incidence", we use the estimated transfer incidence function in the right panel of Figure 6. Much like unequal incidence of labor income – unequal incidence of transfers only has modest effects on amplification.²⁴

4.4 Taylor rule

In the previous section, we studied the sensitivity of our findings to different fiscal adjustment rules to the monetary shock. Here, we analyze sensitivity with respect to to different monetary adjustments. We therefore generalize the Taylor rule (15) to include the output gap

$$i_t = \bar{r}^b + \phi \pi_t + \phi_y \left(\frac{Y_t}{\bar{Y}} - 1\right) + \epsilon_t \tag{21}$$

and set the coefficient on the output gap to 0.25, a standard value in the literature.

Figure plots the IRFs decomposition under the two different Taylor rules. When the monetary authority reacts also to the output gap, the response of the economy is smaller. However, the decomposition between direct and indirect channels remains

²⁴We have also experimented with alternative specifications for the transfer incidence function, with most alternative specifications leading to only modest deviations from the "transfers adjust, equal incidence" case. For example, we have that deviations of transfers from steady state are equally distributed across the entire population (rather than proportionately to steady state transfers as in the "equal incidence" case). This assumption leads to slightly less amplification than in the "equal incidence" case.

roughly unchanged.

5 Conclusions

Heterogeneous Agent New Keynesian (HANK) models contain a number of potential channels that can in principle amplify or dampen the response of aggregate consumption to a monetary policy shock, but which are either absent or less important in Representative Agent New Keynesian models. Our goal in this paper has been to provide some guidance on how a subset of these channels compare in terms of their quantitative strength, in the context of a rich two-asset HANK model, which is calibrated to be consistent with micro evidence on household earnings, wealth distributions and MPCs.

The model elements that we have considered include (i) the presence of capital adjustment costs; (ii) unequal incidence of fluctuations in aggregate labor income across households; (iii) assumptions about the incidence of fluctuations in profits across households and assets; and (iv) assumptions about how fiscal policy reacts in order to maintain intertemporal government budget balance.

Our findings suggest that of these elements, the assumption about which fiscal instruments adjust in response to a monetary expansion is by far the most important. Consistent with previous findings from simpler, analytic HANK models, we find that unequal incidence of labor income, profit income and transfers can all either amplify or dampen the effect of a monetary shock. But for incidence functions implied by our estimates from US data, the degree of amplification or dampening is relatively small.

The amplification or dampening from unequal incidence is even smaller in the presence of capital adjustment costs. This is because although capital adjustment costs do not affect the overall response to a monetary shock, they change its transmission mechanism in a way that makes asset price movements more important than movements in household labor income. The relative amplification or dampening from different fiscal adjustment is not affected by the presence of capital adjustment costs.

Our finding that the fiscal response to a monetary expansion can have powerful effects raises an interesting caveat. In our model, there is full and immediate passthrough of the policy rate set by the Central Bank to the interest rate that the government pays on its debt. This is due to our assumption that all government debt and household assets are of infinitely short duration with an instantaneously adjusting interest rate. In reality, however, a substantial fraction of government debt is long-term, with coupon rates that were set in the past and that do not instantaneously adjust to changes in policy rates. If the government budget constraint were financed with long-term debt then a monetary shock might have a smaller effect on the government budget constraint, and hence assumptions about the fiscal policy reaction might be less consequential.

On the other hand, without additional frictions to generate market segmentation across different parts of the yield curve, a change in short rates would still impact the price of long-term government debt, which in turn would generate additional wealth effects for the households who hold that debt. We view the introduction of long-term government debt and a more serious model of government finances as an under-explored area for future work on HANK models. Such extensions of the HANK framework would represent yet another model element that is potentially important in heterogeneous agent models, but is inconsequential in representative agent models.

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