ONLINE APPENDIX

The Marginal Propensities to Consume in Heterogeneous Agent Models

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This Appendix is organized as follows. Section A describes the estimation of the income process and how we computed some key statistics of the wealth distribution. Section B describes the annual calibration of the baseline model, lays out its continuous time version and derives the MPC under certainty and no borrowing constraints. Section C lays out the household problem under temptation-self control and under present bias. Section D explains how to compute the intertemporal MPCs. Section E contains additional Tables.

A Data

A.1 Panel Study of Income Dynamics

We use data from the Panel Study of Income Dynamics (PSID) on total annual household labor income for households with heads aged 25 to 65 from 1968 to 2008. We drop households with annual labor income less than \$7,250 in 2016 dollars, which correspond to 1,000 hours per year at \$7.25 per hour (or part-time employment at the ongoing minimum wage). We remove age and year effects in a first stage by regressing household labor income on a full set of year and age dummies and we construct the empirical counterparts to $m_{2,d}$ using the residuals from this regression. The resulting moments are shown in the first column of Table A.1.

Let log y_t^{ann} be annual labor income in year *t*, and let annual income growth at lag *d* be

$$
\Delta_d \log y_t^{ann} = \begin{cases} \log y_t^{ann} & \text{if } d = 0\\ \log y_{t+d}^{ann} - \log y_t^{ann} & \text{if } d > 0 \end{cases}
$$

Define cross-sectional moments of annual income growth of order *j* at lag *d* as

$$
m_{j,d} = \mathbb{E}\left[(\Delta_d \log y_t^{ann})^j \right]
$$

$\text{Lag} (d)$	$m_{2,d}$	$m_{4,d}$	κ_d
0	0.504	0.930	3.65
1	0.142	0.220	10.90
$\overline{2}$	0.207	0.369	8.57
3	0.235	0.410	7.42
4	0.280	0.544	6.96
5	0.295	0.557	6.39
6	0.335	0.694	6.19
7	0.352	0.729	5.87
8	0.383	0.838	5.72
9	0.398	0.885	5.59
10	0.422	0.967	5.43

Table A.1: Empirical moments of annual income growth at different lags. Source: PSID 1968- 2008

and the kurtosis of income growth at different lags as

$$
\kappa_d = \frac{m_{4,d}}{\left(m_{2,d}\right)^2}
$$

Table A.1 reports the empirical estimates for the cross-sectional moments that we use in estimation.

A.2 Estimated Income Processes

A.2.1 Discrete Time

We model the discrete-time quarterly income process y_t as follows:

$$
\log y_s = \begin{cases} z_t + \varepsilon_t & \text{with probability } \lambda_{\varepsilon}, \quad \varepsilon_t \sim \mathcal{N}\left(-\frac{\sigma_{\varepsilon}^2}{2}, \sigma_{\varepsilon}^2\right) \\ z_t & \text{with probability } 1 - \lambda_{\varepsilon}. \end{cases}
$$

$$
z_t = \begin{cases} \phi_z z_{t-1} + \eta_t & \text{with probability } \lambda_{\eta}, \quad \eta_t \sim \mathcal{N}\left(-\frac{\sigma_{\eta}^2}{2}, \sigma_{\eta}^2\right) \\ \phi_z z_{t-1} & \text{with probability } 1 - \lambda_{\eta} \end{cases}
$$
(A.1)

We define annual income y^{anm} as the sum of the four quarterly income within the year and, based on this definition, we are able to construct the model counterparts of all the empirical moments in Table A.1.

With λ_{ϵ} , λ_{η} fixed exogenously, we require three moments to estimate the three parameters $\left(\phi_z, \sigma_\eta^2\right)$ *η* , *σ* 2 *ϵ*). Note that the set of moments $\{m_{2,d}\}$ for $d = 1...D$ contains the identical infor-

Process	ϕ_z	σ_n^2	λ_{η}	ϕ_u	σ_{ε}^2	λ_{ε}	$\overline{\sigma_{FE}^2}$
Quarterly							
Baseline	0.988	0.0439	0.250		0.6376	0.250	
Shocks Arrive Quarterly	0.988	0.0108	1		0.2087	1	
Estimated Shock Arrival Rates	0.987	0.0516	0.237		1.6243	0.073	
Krueger, Mitman and Perri (2017 formula)	0.988	0.0108	1		0.0494	1	
Annual							
Baseline	0.953	0.0422			0.0494		
No transitory shocks	0.953	0.0422			0		
High persistence ($\phi_z^{ann} = 0.995$)	0.995	0.0043			0.0688		
With fixed effects	0.916	0.0445			0.0479		0.180
Continuous							
Baseline	0.009	0.134	0.250	0.347	0.652	0.250	
Estimated Shock Arrival Rates	0.012	0.239	0.060	0.347	1.28	0.063	

Table A.2: Parameter estimates of various statistical models for income dynamics.

mation to the auto-covariance function out to *D* lags. We express the data in this way since it is more convenient for extending the estimation strategy to the case where λ_{ϵ} and λ_{η} are also estimated.

In our baseline specification, we choose $(m_{2,0}, m_{2,1}, m_{2,5})$ as our moments to match. The first row of Table A.2 shows our baseline estimates in which we assume that income shocks arrive on average once per year, $\lambda_{\epsilon} = \lambda_{\eta} = 0.25$. In the second row, we shows corresponding estimates when the income shocks arrive every quarter, $\lambda_{\epsilon} = \lambda_{\eta} = 1$. In the third row of Table A.2 we estimate the shock arrival rates λ_{ϵ} , λ_{η} alongside the other parameters of the income process. This requires two additional moments. To find moments that identify these parameters, we note that the main effect of lowering the arrival rates below 1 is that it induces excess kurtosis into the distribution of annual income growth, more a given variance of income growth.

The second and third columns of Table A.1 report *m*4,*^d* and *κ^d* out to ten lags. Note that log income itself does not display much excess kurtosis, but annual income growth is very leptokurtic, with the degree of leptokurtosis declining as the lag length increases. We add κ_1 and κ_5 as the additional moments to identify λ_{ϵ} , λ_{η} . The estimates are reported in the third row of Table A.2. They suggest that persistent shocks arrive on average close to once per year, but that transitory shocks are much less frequent and much larger on average than implied by the more restrictive model.

The fourth row of Table A.2, labeled "Krueger, Mitman and Perri (2017) formula" constructs

the quarterly estimates by applying the following formulas to annual estimates :

$$
\begin{array}{rcl}\n\phi_z & = & (\phi_z^{ann})^{0.25} \\
\sigma_\varepsilon^2 & = & (\sigma_\varepsilon^{ann})^2 \\
\sigma_\eta^2 & = & \left(\frac{1 - \phi_z^2}{1 - (\phi_z^{ann})^2}\right) \left(\sigma_\eta^{ann}\right)^2\n\end{array}
$$

The annual estimates upon which these are based are shown in the bottom panel of Table A.2. These are constructed by reinterpreting *y^t* as annual income and estimating the parameters by matching the moments $(m_{2,0}, m_{2,1}, m_{2,5})$.

In the next section of the table we report estimates for income processes estimated at annual frequency. We first exclude transitory shocks, interpreting them as measurement error. Next, we estimate a version where we restrict the AR(1) component to have high persistence $\phi_z^{ann} = 0.995$, and a version where we include an individual-specific fixed effect. For this latter model we add the moment to $m_{2,10}$ to identify the additional parameter (the variance of the fixed effect σ_{FE}^2).

A.2.2 Continuous Time

We model the continuous time income process y_t as follows:

$$
\log y_t = z_t + u_t
$$

\n
$$
dz_t = -\phi_z z_t + \eta_{it} dJ_{\eta,t}
$$

\n
$$
du_t = -\phi_u u_t + \varepsilon_t dJ_{\varepsilon,t}
$$

where *dJ^η* is a Poisson process with arrival rate *λ^η* and *dJ^ε* is a Poisson process with arrival rate *λε* . The innovations are given by

$$
\eta_{it} \sim N\left(0, \sigma_{\eta}^{2}\right)
$$

$$
\varepsilon_{it} \sim N\left(0, \sigma_{\epsilon}^{2}\right)
$$

Note that since income is a flow, there is no natural concept of purely transitory shock in continuous time. For consistency with the discrete time formulation, so that the two versions match the same data moments, in our baseline model we restrict $\phi_u = \frac{1}{2} \log 2$ which implies a half-life of two quarters. This is broadly consistent with a discrete time annual formulation in which a transitory shocks lasts for one year. In our baseline model we restrict $\lambda_{\eta} = \lambda_{\epsilon} = 0.25$ as in the discrete time model.

The parameter estimates for the continuous time income process are reported in the bottom

panel of Table A.2. We also report estimates for the version where we estimate the shock arrival rates. As in the quarterly discrete time model, when the shock arrival rates are estimated we find them to be larger and less frequent than when restricted to arrive on average once per quarter.

A.3 Survey of Consumer Finances and Wealth Statistics

To compute moments of the wealth distribution, we first select all households in the 2019 *Survey of Consumer Finances*, without any age restriction. Then as explained we drop the top 5% of the wealth distribution.

Our definition of household labor income includes wage and salary income plus social security income. It excludes other business income, other government transfers, as well as interests, dividends and capital gains. Mean household labor income is \$67, 132 and median income is \$54, 266.

Our definition of net worth is the baseline definition of the SCF for total net worth (variable NETWORTH). See the document Networth Flow chart.pdf in [https://www.federalreserve.](https://www.federalreserve.gov/econres/scfindex.htm) [gov/econres/scfindex.htm](https://www.federalreserve.gov/econres/scfindex.htm). It includes all financial assets (bank accounts, CDs, mutual funds, retirement accounts, and directly held stocks and bonds), vehicles, housing wealth and private business equity net of all types of unsecured and secured debt. Mean wealth is \$275, 665 and median wealth is \$103, 380.

Our definition of net liquid and illiquid wealth follows Kaplan and Violante (2014) and Kaplan, Violante, and Weidner (2014). Net liquid wealth includes bank accounts and directly held mutual funds, stocks and bond net of credit card debt. In terms of the SCF variables: FIN - CDS - SACVBND - CASHLI - OTHMA - RETLIQ - (OTHLOC + CCBAL + ODEBT).

Our definition of net illiquid wealth is residual, i.e. net worth minus net liquid wealth. The biggest items among financial assets are retirement accounts, among non-financial assets are housing and business equity. The biggest components on the liability side are mortgages. In terms of the SCF variables net illiquid wealth is: (CDS + SACVBND + CASHLI + OTHMA + RETLIQ) + NFIN - MRTHEL - RESDBT - INSTALL - ODEBT.

B One-Asset Models

B.1 Annual calibration

As in the baseline quarterly calibration, we set $\gamma = 1$, the credit limit to zero, $\delta = 1/50$ so that the expected adult life span is 50 years, and the real interest rate $r = 0.01$. Table A.2 reports the annual value for variances and correlation coefficient estimated to match the same annual covariances restrictions as for the baseline calibration. The discount factor is calibrated internally to match a ratio of mean net worth to mean annual household labor income ratio of 4.1. We obtain an annualized value of 0.980 for the effective discount factor *β*, i.e. virtually the same value as in the quarterly calibration. This is reassuring, since annual and quarterly calibrations should replicate exactly the same set of moments.

B.2 Continuous-time formulation

The continuous-time version of the household problem (1) is:

$$
\max_{\{c_t\}} \mathbb{E}_0 \int_0^\infty e^{-(\delta + \tilde{\rho})t} u(c_t) dt
$$
\n(8.2)
\ns.t.
\n
$$
\dot{b}_t = \exp(y_t) + rb_t - c_t
$$
\n
$$
b_t \ge -\underline{b}
$$
\n
$$
y_t \sim F(y_t, y_{t-1})
$$

In this formulation, $\delta > 0$ is the instantaneous death rate, $\tilde{\rho} > 0$ the discount rate, $\rho = \tilde{\rho} + \delta$, and *b^t* represents savings. The corresponding HJB equation is:

$$
\rho v (b, y) = \max_{c} u (c) + v_{b} (b, y) \dot{b} + A(y) v (b, y)
$$

subject to

$$
\dot{b} = rb + y - c
$$

$$
b \ge 0
$$

where A is the infinitesimal generator of the income process. The continuous time equivalent of the income process in (A.1) is:

$$
y_t = z_t + \varepsilon_t d_{\text{let}},
$$

\n
$$
dz_t = -(1 - \phi) z_{t-1} + \eta_t d_{\eta t}, \text{ with } \eta_t \sim \mathcal{N}(0, \sigma_{\eta})
$$

\n
$$
\varepsilon_t \sim \mathcal{N}(0, \sigma_{\varepsilon})
$$
\n(B.3)

where $J_{\varepsilon t}$ and $dJ_{\eta t}$ are jump processes with arrival rate λ_{ε} and λ_{η} respectively. To estimate the parameters of the income process, we time-aggregate in order to match the same set of annual moments described above. Table A.2 reports the point estimates of the parameters of the income process, expressed quarterly for ease of comparison with the discrete time counterpart.

MPC in continuous time To define and compute the MPC in the continuous time version of the model we follow Achdou, Han, Lasry, Lions, and Moll (2017). In continuous time, the MPC is defined over an interval *τ* as:

$$
\mathfrak{m}_{\tau}(b,y) = \frac{\partial C_{\tau}(b,y)}{\partial b} \simeq \frac{C_{\tau}(b+x,y) - C_{\tau}(b,y)}{x}, \tag{B.4}
$$

where

$$
C_{\tau}(b,y)=\mathbb{E}_0\left[\int_0^{\tau}c(b_t,y_t)dt|b_0=b,y_0=y\right].
$$

The conditional expectation $C_{\tau}(b, y)$ can be conveniently computed using the Feynman-Kac formula. This formula establishes a link between conditional expectations of stochastic processes and solutions to partial differential equations. Applying the formula, we have $C_{\tau}(b, \log y)$ = $K(b, \log y, 0)$, where $K(b, y, t)$ satisfies the partial differential equation on [0, *τ*]

$$
c(b,y) + K_b(b,y,t)\dot{b}(b,y) + K_y(b,y,t) [-(1-\phi) z] + \mathcal{A}(y)K(b,y,t)
$$
 (B.5)

with terminal condition $\Gamma(b, y, \tau) = 0$, where A is the infinitesimal generator of the income process.

B.3 MPC Under Certainty and No Borrowing Constraints

The budget constraint of the household problem (1) is:

$$
c_t = Rb_t + y_t - b_{t+1}
$$

Iterating forward, we obtain:

$$
c_0 + \frac{1}{R}c_1 + \frac{1}{R^2}c_2 + \dots = Rb_0 + \sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^t y_t.
$$

Using the household Euler equation between t and $t + 1$

$$
c_{t+1} = (\beta R)^{\frac{1}{\gamma}} c_t
$$

to substitute c_t at every t on the left hand side as a function of c_0 , we arrive at:

$$
c_0 + \frac{1}{R} c_0 \left(\beta R\right)^{\frac{1}{\gamma}} + \frac{1}{R^2} c_0 \left[\left(\beta R\right)^{\frac{1}{\gamma}} \right]^2 + \dots = R b_0 + \sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^t y_t
$$

and collecting terms on the left hand side:

$$
c_0 \left[\frac{1}{1 - R^{-1} (\beta R)^{\frac{1}{\gamma}}} \right] = R b_0 + \sum_{t=0}^{\infty} \left(\frac{1}{R} \right)^t y_t
$$

which proves that $\mathfrak{m}_0^* = 1 - R^{-1} (\beta R)^{\frac{1}{\gamma}}$.

C Models with Behavioral Biases in Preferences

C.1 Temptation and Self-Control: Discrete Time One-Asset Model

In this Appendix we describe the model of temptation and self-control that we solve in Section 3.2.1. We assume that in each period the agent is tempted to consume its entire wealth and the temptation utility function is the same as for actual consumption. The household problem can then be written in recursive form as:

$$
v(b,y) = \max_{b' \ge 0} \{ u(c) + \beta \mathbb{E}v(b',y') \} + \varphi[u(c) - u(Rb + y)]
$$

subject to

$$
c + b' = Rb + y, \quad b' \ge 0
$$

The parameter $\varphi \geq 0$ measures the strength of the temptation. When $\varphi = 0$, the model collapses to the model without temptation. When *φ* is very large, the agents gives in to temptation and consumes all its cash in hand every period. In this case the MPC out of additional income is one.

The first-order condition for this problem is

$$
u_c(c) = \beta R \mathbb{E}\left[\left(1 - \frac{\varphi}{1 + \varphi} \frac{u_c (R b' + y')}{u_c (c')}\right) u_c (c')\right].
$$
 (C.6)

This first-order condition can be interpreted as a modified Euler equation, with an endogenous discount factor. For example, with log preferences $u(c) = \log(c)$ the endogenous discount factor becomes

$$
\beta\left(1-\frac{\varphi}{1+\varphi}\left(\frac{c'}{Rb'+y'}\right)\right)
$$

which makes it clear that, for a given value of *ϕ*, households who consume a higher fraction of their wealth act as if they are more impatient. These are typically poorer households, and so with this preference formulation, the effective discount factor tends to be lower for household with lower wealth. In the limit, as households become hand-to-mouth, $c' = Rb' + y'$ and their $\text{discount factor becomes } \beta \frac{1}{1+\varphi} < \beta.$

C.2 Temptation and Self-Control: Continuous Time Two-Asset Model

In this Appendix we describe the model of temptation and self-control that we solve in Section 4.3. In continuous time, the recursive formulation of the two-asset model with temptation and self-control can be written as follows.

$$
\rho v (a, b) = \max_{c} (1 + \varphi) u (c) + [\partial_b v (a, b) + \varphi \partial_b \hat{v} (a, b)] (r^b b + y - c)
$$

+
$$
[\partial_a v (a, b) + \varphi \partial_a \hat{v} (a, b)] (r^a a) + A [v + \varphi \hat{v}] (a, b)
$$

+
$$
\lambda [v^* (a, b) - v (a, b)] + \lambda \varphi [\hat{v}^* (a, b) - \hat{v} (a, b)]
$$

-
$$
\varphi \delta \hat{v} (a, b)
$$

where A is the infinitesimal generator of the income process and

$$
v^*(a,b) = \max \left\{ v(a,b), \max_{a'+b'\leq a+b-\kappa} v(a',b') \right\}
$$

The function $\hat{v}(a, b)$ is the temptation value function, which solves

$$
\hat{\rho}\hat{\sigma}(a,b) = \max_{c} u(c) + \partial_{b}\hat{\sigma}(a,b) \left(r^{b}b + y - c\right) + \partial_{a}\hat{\sigma}(a,b) r^{a}a + A\hat{\sigma}(a,b) + \lambda \left[\hat{\sigma}^{*}(a,b) - \hat{\sigma}(a,b)\right]
$$

where

$$
\hat{v}^*(a,b) = \max \left\{ \hat{v}(a,b), \max_{a'+b'\leq a+b-\kappa} \hat{v}(a',b') \right\}
$$

The key assumption is that $\hat{\rho}$ >> ρ , so that the household is tempted to act according to a preference specification that discounts the future at a much higher rate. In our simulations we set the quarterly value for $\hat{\rho} = 90\%$. The first-order condition for consumption satisfies

$$
(1+\varphi) u'(c) = \partial_b v(a,b) + \varphi \partial_b \hat{v}(a,b).
$$

When $\rho = \hat{\rho}$ or $\varphi = 0$, the model collapses to the standard model without temptation.

C.3 Present-Bias

In this Appendix we describe the model of naive present bias that we solve in Section 3.2.2. First consider the problem of a household that does not suffer from present bias:

$$
\rho \tilde{v}(b, y) = \max_{c} u(c) + \tilde{v}_b(b, y) \dot{b} + \mathcal{A}\tilde{v}(b, y)
$$

subject to

$$
\dot{b} = rb + y - c, \quad b \ge 0
$$

where A is the infinitesimal generator of the income process. The first order condition to this optimization problem for $b > 0$ is

$$
u_c(c)=\tilde{v}_b(b,y)
$$

The solution to this problem defines a consumption function given by

$$
\tilde{c}(b,y) = \min\{u_c^{-1}[\tilde{v}_b(b,y)], y\}
$$

A household with naive present bias has a continuation value given by

$$
v(b, y) = \zeta \tilde{v}(b, y) \quad \text{for} \quad \zeta < 1.
$$

So for $b > 0$, consumption solves the first order condition condition

$$
u_c(c) = v_b(b,y) = \zeta \tilde{v}_b(b,y).
$$

With CRRA utility, this gives the consumption function

$$
c(b,y)=\min\{\zeta^{-\frac{1}{\gamma}}\tilde{c}(b,y),y\}.
$$

D MPCs at Different Horizons

To compute the MPC at different horizons, we proceed as follows. Recall the definition of the impact MPC in equation (2) in the main text. Let, for example, $t = 1$ be the horizon of interest. Then, the MPC at horizon 1 out of a windfall income *x* is:

$$
\mathfrak{m}_{1}(x;b,y)=\frac{\int_{Y}\left[c\left(b^{\prime}\left(b+x,y\right),y^{\prime}\right)-c\left(b^{\prime}\left(b,y\right),y^{\prime}\right)\right]dF\left(y^{\prime},y\right)}{x}
$$

Iterating this procedure forward, one obtains $m_t(x; b, y)$, for all $t > 0$. The cumulative MPC until horizon *T* is simply the sum of the MPCs at each horizon $t = 0, 1, ..., T$. The average (or aggregate) MPC at horizon *t* is obtained by integrating the function $m_t(x; b, y)$ under the stationary distribution, i.e.

$$
\bar{\mathfrak{m}}_t(x) = \int_{B \times Y} \mathfrak{m}_t(x; b, y) d\mu(b, y).
$$
 (D.7)

Finally, we are also interested in the MPC out of the news that a windfall of size *x* will be received in the future. For example, the MPC at horizon −1, i.e. out of the announcement that *x* will be paid next period, is:

$$
\mathfrak{m}_{-1}(x; b, y) = \frac{c(x; b, y) - c(b, y)}{x}
$$
 (D.8)

where $c(b, y)$ is the solution to the Bellman equation corresponding to the optimization problem (1) :

$$
v(b,y) = \max_{c} u(c) + \beta \mathbb{E}\left[v(b',y')|y\right]
$$

and $c(x; b, y)$ is the solution to the following Bellman equation, modified to account for the fact that the household expects *x* next period:

$$
v(b,y) = \max_{c} u(c) + \beta \mathbb{E} \left[v(b' + x, y') | y \right]
$$
 (D.9)

and subject to the same set of constraints as (1).

E Additional Tables

Table E.1: Baseline one-asset model and calibrations for different model frequency (annual and continuous time). Model frequency indicates the frequency at which consumption and saving decisions are made. See Table A.2 for details on the income process at different frequencies.

Table E.2: Baseline one-asset model and sensitivity analysis with respect to the statistical process for income dynamics. The columns correspond, respectively, to income processes whose parameters are in lines (1), (6), (3), (4), (2), (7) and (8) of Table A.2.

Table E.3: Baseline one-asset model and sensitivity analysis with respect to survival rates and to assumptions on how assets of the deceased are distributed among the living. In the baseline, everyone starts with zero wealth. In the model with bequest, assets of the deceased are distributed equally to the newborn. In the model with no death $(\delta = 0)$ households have an infinite horizon. The last specification has perfect annuity markets.

Table E.4: One-asset model with heterogeneity in the curvature parameter γ of the CRRA utility function.

Table E.5: One-asset model with Epstein-Zin preferences. RRA: coefficient of relative risk aversion (*γ*). IES: intertemporal elasticity of substitution (1/*θ*). See equation (4).

Table E.6: One-asset model with preferences featuring present bias in consumption choices. The parameter $\zeta < 1$ measures the strength of the present bias (the baseline model without present bias features $\zeta = 1$).

Table E.7: Two-asset baseline model and sensitivity with respect to the rebalancing frequency *χ* and the transaction cost *κ*.